Weaving frames linked with fractal convolutions

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1 Abstract

Weaving frames have been introduced to deal with some problems in signal processing and wireless sensor networks. The notion of fractal operator and fractal convolutions have been linked with perturbation theory of Schauder bases and frames. However, the existing literature has established limited connections between the theory of fractals and frame expansions. More recently, the fractal perturbation have been recognized as a binary operation between two functions, namely the germ function and the base function (aside from other elements such as partition and scale factors). The aforementioned binary operation is called a fractal convolution. In this paper, we define Weaving frames generated via fractal operators combined with fractal convolutions. The aim is to demonstrate how partial fractal convolutions are associated to Riesz bases, frames and the concept of Weaving frames.

Two frames $\{\phi_i\}_{i\in I}$ and $\{\psi_i\}_{i\in I}$ for a separable Hilbert space $\mathcal{H}$ are said to be woven, if there are universal positive constants A and B such that for every subset $\sigma \subset I$, the family $\{\phi_i\}_{i\in\sigma}\cup\{\psi_i\}_{i\in\sigma^c}$ is a frame for $\mathcal{H}$ with lower and upper frame bounds A and B, respectively. We study conditions in which new bases and weaving frames can be obtained from the old ones by using the algebraic operation of fractal convolutions. This allows us to construct frames and bases consisting of self-referential functions, which may find potential applications in areas such as differential equations and signal processing. Preservation of basis property by fractal convolutions with null function is studied, which calls for the pursuance of a somewhat different tack.

On the one hand, the distinctiveness of fractals and fractal functions is claimed by the fractal researchers, and on the other, as observed, there are only a few results describing the algebraic properties of the fractal functions such as their ability to span important function spaces. The current note may also be viewed as a further contribution to bridge this gap. The view point deals with ones sided convolutions i.e both left and right partial fractal convolution operators on Lebesgue space L^p $(1 \leq p < \infty)$. Some applications via partial fractal convolutions with null function have been obtained for the perturbation theory of bases and weaving frames.

References

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