

# Regularity results for classes of Hilbert $C^*$ -modules w.r.t. special bounded modular functionals

Michael Frank

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## Hilbert C\*-modules and Gabor analysis

Starting about 2009 Franz Luef and later his group at Trondheim (Norway) have given a **reformulation of the duality principle of Gabor analysis to the setting of Morita equivalence bimodules.**

The duality theory for module frames for equivalence bimodules between Morita equivalent C\*-algebras in a specialized setting has been investigated, and it appears to be a true generalization of the duality theory.

A. Austad, U. Enstad, M. S. Jakobsen, F. Luef, D. Bakić.

## Hilbert C\*-modules

Let  $A$  be a  $C^*$ -algebra and  $M$  be a complex-linear space and (right)  $A$ -module where both the complex-linear structures are compatible.

Let  $M$  admit a map  $\langle \cdot, \cdot \rangle: M \times M \rightarrow A$  conjugate- $A$ -linear in the first and  $A$ -linear in the second variable such that

- (i)  $\langle x, y \rangle = \langle y, x \rangle^*$  for any  $x, y$  in  $M$ ,
- (ii)  $0 \leq \langle x, x \rangle$  for any  $x$  in  $M$ ,
- (iii)  $\langle x, x \rangle = 0$  iff  $x=0$ .

Then  $\|x\| := \|\langle x, x \rangle\|_A^{1/2}$  defines an  $A$ -module norm on  $M$ .

*Examples .*

Motivated by the ways of derivation of sub-Hilbert product systems from Hilbert product systems O. M. Shalit and B. Solel, later M. Skeide rised the following new question:

**Question:**

Assume, there are two Hilbert  $A\$$ -modules  $M \subset N$  such that  $M^\perp = \{0\}$ .

Might there exist a non-trivial bounded  $A$ -linear map  $r_0: N \rightarrow A$  such that  $r_0$  vanishes on  $M$ ?

In other words, might  $M$  and  $N$  be separated by a non-trivial bounded  $A$ -linear map  $r_0: N \rightarrow A$  such that  $r_0$  vanishes on  $M$ ?

With some work one obtains a consequence:

**Theorem:**

Let  $A$  be a  $C^*$ -algebra and  $M, N$  be two Hilbert  $A$ -modules such that  $M \subset N$  with  $M^\perp = \{0\}$ . Then

there exists a non-zero bounded  $A$ -linear map  $r_0: N \rightarrow A$  that is equal to zero on  $M$

iff

there exists a bounded  $A$ -linear map  $T_0: N \rightarrow N$  such that  $\text{Ker}(T_0) \supsetneq M$  is unequal to  $\text{Ker}(T)^{\perp\perp}$ .

In 2021 M. Skeide and J. Kaad were able to construct a (complicated) counter-example, see arxiv / JOT.  
 $A=C([0,1])$  ,  $N=I_2(A)$  with existing  $r_0$ .

**Known answers:**

For  $A=\mathbb{C}$  and, thus, Hilbert spaces the answer is *no*.

For  $A=K(I_2)$  on the Hilbert space  $I_2$  and resp. Hilbert  $A$ -modules the answer is *no*.

For modular maximal right ideals  $D$  of  $C^*$ -algebras  $A$ ,  $A$  without finite part, the answer is *no* (non-trivial, since left actions, finite part different).

Very difficult to find new examples ... not understood.

## Intuition says:

The answer might be *no* for  $C^*$ -algebras  $A$  of coefficients that are von Neumann algebras, monotone complete  $C^*$ -algebras or compact  $C^*$ -algebras, e.g..

So, there one has to find a proof or some more counter-examples – and to understand them!

The situation with norm-closed right ideals is more complicated, generally speaking. Non-commutative topology is not well-behaved ... not all maximal norm-closed right ideals might be modular ...

**Lemma:**

Let  $A$  be a  $C^*$ -algebra and  $M \subseteq N$  be two full Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ . Then:

- (i) Two different elements  $n_1, n_2$  in  $N$  restricted to  $M \subseteq N$  realize pairwise distinct bounded  $A$ -linear functionals  $\langle n_1, . \rangle, \langle n_2, . \rangle$  on  $M$ .
- (ii) In case of the existence of a non-zero bounded  $A$ -linear functional  $r_0: N \rightarrow A$  such that  $r_0$  vanishes on  $M$  the entire module  $\{ r_0 a : a \in A \}$  as well as its norm-closure in  $N'$  represents the zero functional on  $M$ . So  $r_0$  cannot be represented by an element of  $N$  via the  $A$ -valued inner product on  $N$  by supposition.

- (iii) The Hilbert A-module  $N$  admits (at least) two A-valued inner products inducing equivalent norms, whose reductions to  $M$  coincide. The more, these two A-valued inner product are not equivalent, i.e.  
 $\langle T(\cdot), \cdot \rangle^{\{(1)\}} \neq \langle \cdot, \cdot \rangle^{\{(2)\}}$  for any positive w.r.t.  $\langle \cdot, \cdot \rangle^{\{(1)\}}$ , bounded bijective A-linear operator  $T$  on  $N$ .
- (iv) Given the situation at (iii), the notions of bounded module operators on  $N$  to be "compact" or to be adjointable w.r.t. one of these two named A-valued inner products depend on the choice of the A-valued inner product on  $N$  with identical reductions to  $M$ .

**Lemma:**

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two full Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ . Then:

- (i) The  $C^*$ -algebras  $\langle M, M \rangle$  and  $\langle N, N \rangle$  in  $A$  have the same central carrier projection  $p$  in  $A$ , i.e. both their annihilators with respect to  $A$  equal  $(1-p)A$  with  $p$  in  $Z(A)$ . Obviously, both  $\langle M, M \rangle$  and  $\langle N, N \rangle$  are two-sided norm-closed ideals of the monotone complete  $C^*$ -algebra  $pA$ , as well as  $\langle M, M \rangle$  is a two-sided ideal of  $\langle N, N \rangle$  in  $pA$ .

- .....
- (ii) The multiplier  $C^*$ -algebras of their centers equal to  $pZ(A)$ , so the unitizations of  $\langle M, M \rangle$  and of  $\langle N, N \rangle$  inside  $pA$  share the identity element  $p$  of  $pZ(A)$  in  $pA$  as their respective identities.
  - (iii) The multiplier  $C^*$ -algebras of both  $\langle M, M \rangle$  and  $\langle N, N \rangle$  equal to  $pA$ , so the centers of their multiplier  $C^*$ -algebras both equal to  $pZ(A)$ , too. (In general,  $Z(\mathbf{M}(A))$  can be larger than  $\mathbf{M}(Z(A))$ .) So the two  $C^*$ -algebras  $\langle M, M \rangle$  and  $\langle N, N \rangle$  share the identity element  $p$  in  $pZ(A)$  of their unitizations even in this sense.
  - (iv) The centers of the  $C^*$ -algebras  $\text{End}_A^*(M)$  and  $\text{End}_A^*(N)$  of all (bounded) adjointable  $A$ -linear operators on  $M$  and on  $N$ , respectively, are isometrically  $*$ -isomorphic to  $pZ(A) \subseteq pA$ .

**Facts:** W. L. Paschke // M. Hamana / M. Frank

Let  $A$  be a  $W^*$ -algebra vs. a monotone complete  $C^*$ -algebra. Let  $M$  be a Hilbert  $A$ -module with an  $A$ -valued inner product  $\langle \cdot, \cdot \rangle$ ,  $x$  in  $M$ .

Let  $M'$  be the Banach  $A$ -module of all bounded  $A$ -linear maps  $r$  from  $M$  to  $A$ .

Then the  $A$ -valued inner product can be uniquely continued to an  $A$ -valued inner product on  $M'$  preserving the isometric embedding  $M \subseteq M'$  and the values  $r(x) = \langle r, x \rangle$  and  $\langle r, r \rangle = \sup \{ r(x)^* r(x) : \|x\| \leq 1 \}$ .

As Banach modules always:  $M \subseteq M'' \subseteq M'$  and  $M' = M''$ .

In the  $W^*$ -case vs. monotone complete case:  $M' = M''$ .

**Theorem:** (cf. [25, Thm. 13])

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

Then the selfdual Hilbert  $A$ -module  $M'$  admits an isometric embedding as a Hilbert  $A$ -submodule of the selfdual Hilbert  $A$ -module  $N'$ , which extends the given isometric embedding of  $M$  in  $N$  in the same way as  $M$  is isometrically embedded in  $M'$ .

The embedded copy of  $M'$  in  $N'$  is an orthogonal direct summand,  $P: N' \rightarrow M'$ .

*Equal???*

No more information on the module level !?

**Idea:** Switch to the consideration of full Hilbert A-modules  $N$  as  $K_A(N)$ -A bimodules in the sense of strong Morita equivalence of  $A$  and of  $K_A(N)$ .

**Note:**  $A \equiv K_{K_A(N)}(N)$ , so there is some symmetry in the considerations.

**Use:**  $\text{End}_A^*(N) \equiv M(K_A(N))$       \*-isometrically  
 $\text{End}_A(N) \equiv LM(K_A(N))$       isometrically

**Consider:**  $M \subseteq N$  ,  $M \subseteq M'$  ,  $N \subseteq N'$  ,  $M \subseteq N'$

**Lemma:**

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

Then any orthogonal, positive projection  $P: N' \rightarrow N'$  with  $P \neq \text{id}_N$  and  $M' \subseteq P(N')$  is not an element of the multiplier  $C^*$ -algebra  $\text{End}_A^*(N)$  of the  $C^*$ -algebra of "compact" module operators  $K_A(N)$  of the isometrically embedded in  $N'$  copy of  $N$ , i.e. any such  $P \neq \text{id}_N$  with  $M' \subseteq P(N')$  is not a bounded adjointable (module) operator on  $N$ .

## Proposition:

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

The four operator norms  $\|T\|_M$ ,  $\|T\|_N$ ,  $\|T\|_{M'}$  and  $\|T\|_{N'}$  coincide for any "compact" operator  $T$  in  $K_A(M)$  realized on resp.  $M$ ,  $N$ ,  $M'$  and  $N'$ . As a consequence, the given isometric modular embedding  $M \subseteq N$  gives rise to an isometric  $*$ -representation of  $K_A(M)$  in  $K_A(N)$  on the level of "compact" modular operators on  $M$  and on  $N$ , respectively.

## Proposition:

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

Then  $\text{End}_A^*(N)$  does not contain any non-zero element  $T$  such that  $T$  is perpendicular to  $K_A(M) \subseteq K_A(N)$ .

Then the identity operator on  $M$  extends to the identity operator on  $N$ . This is its unique extension as a bounded modular adjointable operator which preserves the norm.

**Theorem:** (cf. [25, Thm. 9, Thm. 10], commutative case / type I von Neumann case)

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

Then the selfdual Hilbert  $A$ -module  $M'$  admits an isometric embedding as a Hilbert  $A$ -submodule of the selfdual Hilbert  $A$ -module  $N'$  such that  $M'$  coincides with  $N'$  preserving the given isometric inclusions. In particular, there does not exist any bounded  $A$ -linear functional  $r_0: N \rightarrow A$  such that  $r_0$  vanishes on  $M$ , but which is not the zero functional on  $N$ .

**Corollary:** (cf. [25, Thm. 10])

Let  $A$  be a monotone complete  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert  $A$ -modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to  $N$ .

Then the  $A$ -dual Banach  $A$ -modules of  $N$  and of  $M$  isometrically coincide as Banach  $A$ -modules. In particular, for a given Hilbert  $A$ -module  $N$  this holds for any (smaller-equal) Hilbert  $A$ -submodule  $M$  with  $M^\perp = \{0\}$ .

**Theorem:**

The kernel of any bounded A-linear operator between two Hilbert A-modules over a monotone complete C\*-algebra A is biorthogonally complemented.  
(I.e. not only for bounded *adjointable* operators T.)

**Corollary:**

Let A be a monotone complete C\*-algebra and D be a right, norm-closed ideal of A which is order dense in A. Then there does not exist any non-zero bounded A-linear functional  $r_0: A \rightarrow A^*$  such that  $r_0(D) = \{0\}$ .

A – compact  $C^*$ -algebra, i.e. A admits a faithful  $*$ -representation in some  $C^*$ -algebra of compact operators on a suitable Hilbert space

**Theorem:** (cf. [25, Thm. 10])

Let A be a compact  $C^*$ -algebra and  $M \subseteq N$  be two Hilbert A-modules. Suppose that  $M \subseteq N$  has the orthogonal complement  $M^\perp = \{0\}$  with respect to N.

Then the bounded A-linear zero functional of M to A admits a unique extension to a bounded A-linear functional of N to A such that it vanishes on M – the zero functional (because  $M=N$ ). The more, the kernel of every bounded A-linear operator mapping it to another Hilbert A-module is biorthogonally completed in it.  
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**Theorem:**

Let  $A$  be a  $C^*$ -algebra and  $D$  be a right modular maximal ideal in  $A$ . Consider them as Hilbert  $A$ -modules in the usual way transferring the algebraic properties of  $A$  to modular and  $A$ -valued inner product structures on both  $A$  and  $D \subset A$ . Then the zero functional on the Hilbert  $A$ -submodule  $D$  has only the zero modular functional on its biorthogonal complement  $D^{\perp\perp}$  in the hosting Hilbert  $A$ -module  $A$  as its continuation.

modular ideals = there exists an element  $u_D$  in  $A$  such that  $(a - u_D a)$  in  $D$  for any  $a$  in  $A$  – they are automatically norm-closed, cf. Murphy, Thm. 1.3.1.

Every maximal right ideal of a  $C^*$ -algebra is norm closed, cf. CGDRP, Cor. 3.6.

Every maximal right ideal of a Fréchet algebra is modular, cf. Davis, Thm. 2.2.42, Prop. 4.10.23.

In the commutative  $C^*$ -case the codimension is 1 CGDRP, Thm. 2.4(i), Gel'fand-Mazur theorem; arbitrary large in the non-commut. case, cf. CGDRP.<sup>22</sup>

**Corollary:**

Let  $A$  be a monotone complete  $C^*$ -algebra. Let  $I$  and  $J$  be two right norm-closed ideals of  $A$  such that  $I \subset J \subseteq A$  with the orthogonal complement  $pA$ ,  $p=p^2 \geq 0$  in  $A$ . Switching to a consideration of  $I$  and  $J$  as Hilbert  $A$ -submodules of  $A$ , the orthogonal complement of  $I$  with respect to  $J$  is equal to  $\{0\}$ .

Then the right ideals  $I \subset J$  have the same left carrier projection  $(1-p)$  in  $A$ , and as Hilbert  $A$ -submodules of  $A$  they fulfil  $I' = J' = (1-p)A$ . In particular,  $I$  and  $J$  are order dense in  $(1-p)A$ , or equivalently in the picture of Hilbert  $A$ -modules,  $T_2^0$ -dense. So, there does not exist any bounded  $A$ -linear functional  $r_0: J \rightarrow A$  such that  $r_0$  vanishes on  $I$ , but which is not the zero functional on  $J$ .

## Proposition:

Let  $A$  be a commutative  $C^*$ -algebra and  $I \subseteq J$  be two essential norm-closed ideals of  $A$ . Then:

The left and the right annihilator sets of  $I$  w.r.t.  $J$  and of  $I$  or  $J$  w.r.t.  $A$  consist only of the zero element.

The multiplier algebra of  $J$  is isometrically  $*$ -algebraically represented in the multiplier algebra of  $I$ , i.e.  
 $\mathbf{M}(J) \subseteq \mathbf{M}(I)$ .

Suppose,  $I$  and  $J$  are considered as (right) Hilbert  $A$ -modules. Then any bounded  $A$ -linear functional from  $J$  to  $A$  which is supposed to be the zero functional on  $I$  equals to zero on  $J$ .

## Open Questions:

- (0) Understand the structural nature of counter-examples.  
Find more of them.
- (i) Prove the last Prop. for all essential two-sided ideals in  $C^*$ -algebras.
- (ii) Consider one-sided norm-closed ideals of  $C^*$ -algebras and revisit non-commutative topology.
- (iii) Revisit types of strict closures of ideal Hilbert  $C^*$ -modules (i.e. those pairs) (D. Bakić and B. Guljaš)
- (iv) Develop (un)bounded operator theory on Hilbert  $C^*$ -modules beyond adjointable ones.
- (v) Understand the structural nature of orthogonality ...

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HOCHSCHULE FÜR TECHNIK, WIRTSCHAFT UND KULTUR (HTWK) LEIPZIG, FAKULTÄT INFORMATIK UND MEDIEN, PF 301166, D-04251 LEIPZIG, GERMANY.

*E-mail address:* michael.frank@htwk-leipzig.de



## Discussions ...

# Thank you for your attention.

## 1. ABC

### 2. t

#### 1.1. Unterüberschrift

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- **Aufzählungsliste Beispiel A**
  - Unteraufzählungsliste Beispiel I
  - Unteraufzählungsliste Beispiel II
    - Unterunteraufzählung 1
    - Unterunteraufzählung 2