

# Completeness of shifted dilates in invariant Banach spaces of tempered distributions

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ANUPAM GUMBER

*NuHAG, Faculty of Mathematics  
University of Vienna, Vienna, Austria  
anupam.gumber@univie.ac.at*

We show that well-established methods from the theory of Banach modules and time-frequency analysis allow to derive completeness results for the collection of shifted and dilated version of a given (test) function in a quite general setting. While the basic ideas show strong similarity to the arguments used in a recent paper by V. Katsnelson we extend his results in several directions, both relaxing the assumptions and widening the range of applications. There is no need for the Banach spaces considered to be embedded into  $(L^2(\mathbb{R}), \|\cdot\|_2)$ , nor is the Hilbert space structure relevant. We choose to present the results in the setting of the Euclidean spaces, because then the Schwartz space  $\mathcal{S}'(\mathbb{R}^d)$  ( $d \geq 1$ ) of tempered distributions provides a well-established environment for mathematical analysis. We also establish connections to modulation spaces and Shubin classes, showing that they are special cases of Katsnelson's setting. This is a joint work with Hans G. Feichtinger.