SPACES OF VARIABLE BANDWIDTH AND SIGNAL RECONSTRUCTION

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Abstract. A function $f \in L^2(\mathbb{R})$ is said to have bandwidth $\Omega > 0$, if Ω is the maximal frequency contributing to f. The concept of variable bandwidth arises naturally and it is particularly intuitive when considering music, where the highest frequency (the note) varies with time. However, producing a rigorous definition of variable bandwidth is a challenging task, since bandwidth is global by definition and the assignment of a local bandwidth meets an obstruction in the uncertainty principle. In this contribution, we present a new approach to the study of spaces of variable bandwidth via Wilson basis [1, 2]. Our idea is to start with a discrete time-frequency representation that allows us to represent any function as a series expansion of time-frequency atoms with a clear localization both in time and frequency. We may then prescribe a time-varying frequency truncation and, in this way, end up with a space of a given variable bandwidth. For these spaces, we study sufficient conditions and necessary density conditions for sampling. Moreover, analyzing some numerical experiments, we motivate why these new spaces could be useful for the reconstruction of particular classes of functions.
The presentation is based on a joint work with Karlheinz Gröchenig.

References

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[2] B. Andreolli, K. Gröchenig. *Variable bandwidth via Wilson bases*. Appl. Comput. Harmon. Anal., doi: 10.1016/j.acha.2024.101641. 2024.

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