Sampling projections in the uniform norm

Kateryna Pozharska

Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine,
Faculty of Mathematics, Chemnitz University of Technology, Germany

The theorem of Kadets and Snobar [1] asserts that for any n-dimensional subspace V_n of a normed space G, there is a linear projection $P\colon G \to V_n$ with $\|P\| \leq \sqrt{n}$. However, it is not clear what *information* of a function $f \in G$ is required to compute its projection.

We consider the case (see [2] for details), when only *function evaluations* of f are allowed as information, and therefore restrict to $G = L_\infty(D)$, i.e., the space of all bounded complex-valued functions on a set D equipped with the sup-norm $\|f\|_\infty := \sup_{x\in D} |f(x)|$.

In the talk, I will show that for any $V_n \subset L_\infty(D)$ there are $2n$ points $x_1, \ldots, x_{2n} \in D$ and functions $\varphi_1, \ldots, \varphi_{2n}$ such that $Pf = \sum_{i=1}^{2n} f(x_i)\,\varphi_i$ is a projection with $\|P\| \leq C\sqrt{n}$, where $C > 0$ is an absolute constant.

The result is based on a specific type of discretization of the uniform norm, connected to the Marcinkiewicz–Zygmund inequalities. In particular, we get that there are $2n$ points $x_1, \ldots, x_{2n} \in D$ such that $\|f\|_\infty \leq C \left(\sum_{k=1}^{2n} |f(x_k)|^2\right)^{1/2}$ for every $f \in V_n \subset L_\infty(D)$.

Further, we discuss consequences for optimal recovery and new sharp bounds for the n-th linear sampling numbers.

References

[1] M. I. Kadets and M. G. Snobar, *Some functionals over a compact Minkovskii space*, Mathematical Notes, 10(4):694–696, 1971.

[2] D. Krieg, K. Pozharska, M. Ullrich, T. Ullrich, *Sampling projections in the uniform norm*, arXiv: 2401.02220, 2024.

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