Pointwise Multipliers for Besov Spaces with $0 < p \leq 1$

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In my talk I will discuss characterizations of the space of all pointwise multipliers $M(B^s_{p,p}(\mathbb{R}^n))$ for Besov spaces $B^s_{p,p}(\mathbb{R}^n)$ with $0 < p \leq 1$ and $s \in \mathbb{R}$.
Starting point will be a characterization of $M(B^0_{1,1}(\mathbb{R}^d))$ due to Y. Meyer in 1992. Meyer's investigations have been continued by Balazs, Gröchenig and Speckbacher in 2019. These authors have dealt with the cases $M(B^s_{1,1}(\mathbb{R}^n))$, $s \in \mathbb{R}$. Both references provide characterizations in terms of wavelet coefficients. In a first step we have extended the used arguments to the following more general situation: $M(B^s_{p,p}(\mathbb{R}^n))$, $0 < p \leq 1$, $s \in \mathbb{R}$. But the main progress we have obtained is the change from wavelet-type characterizations to Fourier analytic characterizations. This will be discussed in detail. Whereas the wavelet-type characterization of $M(B^s_{p,p}(\mathbb{R}^n))$, $0 < p \leq 1$, $s \in \mathbb{R}$ is in some sense uniform, this is not true for the Fourier analytic characterizations. To our own surprise this program requires a splitting of the parameter domain $(0, 1] \times \mathbb{R}$ into six subdomains. Here it is also interesting to notice that in three cases Besov-type spaces (smoothness spaces related to Morrey spaces) are part of the characterization.
This is joined work with Yinqin Li, Dachun Yang and Wen Yuan (Beijing Normal University).