Sampling in (quasi) shift-invariant spaces generated by ratios of exponential polynomials

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Given a separated set Γ in $\mathbb{R}$ and a generator (function) g. The quasi shift-invariant space $V^p_\Gamma(g)$ generated by function g consists of all functions of the form

$$f(x) = \sum_{\gamma\in\Gamma} c_\gamma g(x-\gamma), \quad \{c_\gamma\} \in l^p(\Gamma).$$

This space is called shift-invariant if $\Gamma = \mathbb{Z}$.

We introduce two families of generators that admit meromorphic extension to the complex plane:

$$\mathcal{C}(\alpha) = \left\{\frac{P(e^{\alpha x})}{Q(e^{\alpha x})}\right\}, \quad \mathcal{K}(\alpha) = \left\{\frac{e^{-\alpha x^2/2}P(e^{\alpha x})}{Q(e^{\alpha x})}\right\}, \quad x\in\mathbb{R},$$

where $\alpha > 0$ and P, Q are polynomials satisfying some natural assumptions.

We will discuss sharp sampling theorems for the quasi shift-invariant spaces $V^p_\Gamma(g), g\in\mathcal{C}(\alpha)$, and shift-invariant spaces $V^p_\mathbb{Z}(g), g\in\mathcal{K}(\alpha)$. Our results are given in terms of lower and upper densities of the sampling set Λ and the set of translates Γ. As an application, we solve the interpolation problem in the space $V^p_\Gamma(g)$ and obtain new results on the density of semi-regular lattices of Gabor frames

$$\mathcal{G}(g,\Lambda,\mathbb{Z}) = \{e^{2\pi int}g(t-\lambda) : \lambda\in\Lambda, n\in\mathbb{Z}\}$$

generated by $g\in\mathcal{C}(\alpha)\cup\mathcal{K}(\alpha)$.

The talk is based on joint work with Alexander Ulanovskii.

References

[1] Alexander Ulanovskii, Ilya Zlotnikov, *Sampling in quasi shift-invariant spaces and Gabor frames generated by ratios of exponential polynomials*,
preprint: arxiv.org/abs/2402.03090